233\textsuperscript{U} is supplied to a moderated Molten Salt Reactor (MSR) for initial inventory and for makeup fuel that comes from a fusion breeder with up to 5% but typically 2.4\% \textsuperscript{232}U to aid nonproliferation. The equilibrium \textsuperscript{232}U concentration builds up above the input 2.4\% for the conversion ratio (CR) less than \( \frac{3}{4} \) and is less than 2.4 for CR above \( \frac{3}{4} \). We can augment the \textsuperscript{232}U by supplying \textsuperscript{231}Pa from the fusion breeder that by neutron capture produces \textsuperscript{232}U in situ. This is important when the MSR produces most of its make up fuel (CR>\( \frac{3}{4} \)) with too little \textsuperscript{232}U to meet the IAEA standard for “self-protection” of 2.4\% \textsuperscript{232}U/U. For breakeven breeding (CR=1) where just as many fissile atoms are produced as are destroyed, we need 6.4 kg of \textsuperscript{231}Pa per full power year per 1000 MWe. Alternatively, 6.4 kg of \textsuperscript{230}Th per full power year per 1000 MWe can supply the needed \textsuperscript{231}Pa.

**Background and introduction**

We treat two moderated MSR designs, a one-fluid and a two-fluid design. The one-fluid design such as the Molten Salt Breeder Reactor (MSBR)\textsuperscript{1} and Denatured Molten Salt Reactor (DMSR)\textsuperscript{2} consists of a pot with a mixture of molten salt pumped through graphite. The size and power are flexible. The chemistry of separations of built up fission products is complicated because many fission products have chemistry similar to that of thorium. The two-fluid design has one fluid in one region containing fissile material to maintain the chain reaction and another region with a second molten salt whose purpose is to capture leakage neutrons in the fertile material, \textsuperscript{232}Th to breed fissile material. Simpler chemical separations are possible without thorium in the fissile region, however, there is a neutron damage issue with the material separating the two regions and the two fluids and the power is limited in some designs in order to maintain high leakage of neutrons into the second region for adequate breeding.

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The two-fluid design has a proliferation problem when its conversion of fertile to fissile material approaches the fissile consumption rate in the fissile zone that is for CR>3/4. The $^{231}$Pa imported from the fusion breeder and put in the fertile zone produces enough $^{232}$U to be 2.4% of the $^{233}$U produced so is self-protected. However, and here lies the problem, the $^{232}$U burns out in the fissile region more slowly than the $^{233}$U is fissioned so as to make the $^{232}$U/$^{233}$U ratio far exceed 2.4%, hence over kill. The two-fluid MSR has this unsolved proliferation issue that needs a solution before it can be promoted. This problem is described briefly towards the end of this note.

**One-fluid MSR**

We construct a simple model of the molten salt reactor in order to compute the amount of $^{232}$U and $^{231}$Pa needed to satisfy the “self-protection” criteria of $^{232}$U/$^{233}$U>2.4%, that is that dose rate from 5 kg sphere of $^{233}$U one year after chemical separation will be 100 rem/h (1 Sv/hr) at one meter.$^3$

$M_1$ is the mass of $^{231}$Pa,
$M_2$ is the mass of $^{232}$U,
$M_3$ is the mass of $^{233}$U,
$M_5$ is the mass of $^{235}$U,
$M_9$ is the mass of $^{239}$Pu.

The fissile in the reactor core and primary loop including processing inventory is assumed to be a constant in our simple model, typically 1500 kg for 1000 MWe. MSBR had 1500 kg $^{233}$U of which 62% was in the core. Its average and peak core power density is 22 and 30 MW/m$^3$; the graphite had a void fraction filled with molten salt of 0.37 in the center and 0.13 in the outer zones; the volume of molten salt in the primary system was 49 m$^3$ with 30 m$^3$ in the vessel (19 m$^3$ in the loop outside the core). DMSR had an average of 3440 kg fissile of which 1970 kg $^{233}$U and 1020 kg $^{235}$U of which 80% was in the core. Its average power density is reduced to 2.9 MW/m$^3$ in order to reduce graphite radiation damage rate; the graphite had a void fraction filled with molten salt of 0.2 in the center and 0.13 in the outer zones; the volume of molten salt in the primary system was 104 m$^3$ with 84 m$^3$ in the vessel (20 m$^3$ in the loop outside the core).

$\dot{M}_{1_0}$ is the mass of $^{231}$Pa supplied yearly from the fusion breeder.
$\dot{M}_{2_0}$ is the mass of $^{232}$U yearly supplied externally from the fusion breeder.
$\dot{M}_{3_0}$ is the mass of $^{233}$U yearly supplied externally. The amount of fissile consumed is typically 1000 kg/y by fission and 90 kg/y that is lost by capture reactions.

\[ \dot{M}_{36} = 1090 \text{ kg } ^{233}\text{U} / \text{y} \cdot (1 - CR) \]

CR = conversion ratio of \(^{233}\text{U}\) produced from thorium to fissile consumed.

The cross sections of interest for thermal neutrons for our reactor with lots of graphite:

\[ \sigma_{\text{absorption}} = 200 \text{ b (n,}\gamma\text{)} \text{ and } 0.02 \text{ b (n,}f\text{)} \text{ for } ^{231}\text{Pa, b for barns in units of } 10^{-24} \text{ cm}^2 \]

\[ \sigma_{\text{absorption}} = 73 \text{ b (n,}\gamma\text{)} \text{ and } 75 \text{ b (n,}f\text{)} \text{ for } ^{232}\text{U} = 148 \text{ b} \]

\[ \sigma_{\text{absorption}} = 531 \text{ b (n,}f\text{)} \text{ and } 46 \text{ b (n,}\gamma\text{)} \text{ for } ^{233}\text{U loss} = 577 \text{ b} \]

\[ \sigma_{\text{absorption}} = 586 \text{ b (n,}f\text{)} \text{ and } 186 \text{ b (n,}\gamma\text{)} \text{ for } ^{235}\text{U loss} = 772 \text{ b} \]

\[ \sigma_{\text{absorption}} = 750 \text{ b (n,}f\text{)} \text{ and } 271 \text{ b (n,}\gamma\text{)} \text{ for } ^{239}\text{Pu loss} = 1021 \text{ b} \]

In order to maintain criticality, the amount of fissile material must be constant. This is true only in our simple model with no fission products building up to absorb neutrons and no other fissile material than \(^{233}\text{U}\) or if we use \(^{235}\text{U}\) or \(^{239}\text{Pu}\) they are assumed to behave the same in our simple model.

We simplify the model of the fission reactor by assuming 1000 MWe (2500 MW\text{nucler} @ 40\%) with thorium so that the steady state inventory is 1500 kg of \(^{233}\text{U}\) independent of what ever fissile is used to start up. We assume the reaction rates can be described with the constant c\(\Phi\) independent of what the fissile material is.

The following equation governs the concentrations of \(^{233}\text{U}\), \(^{235}\text{U}\) and \(^{239}\text{Pu}\).

\[ \frac{dM_5}{dt} = -cM_5 \cdot (\sigma_{\text{fission}}^5 + \sigma_\gamma^5) \cdot \Phi \]

\[ M_5(t \rightarrow \infty) = 0 \]

\[ \frac{dM_9}{dt} = -cM_9 \cdot (\sigma_{\text{fission}}^9 + \sigma_\gamma^9) \cdot \Phi \]

\[ M_9(t \rightarrow \infty) = 0 \]

Since we assume an initial inventory but no source of \(^{235}\text{U}\) and \(^{239}\text{Pu}\); make up is always assumed to be \(^{233}\text{U}\).

The following equation governs the concentrations of \(^{233}\text{U}\).
\[ \frac{dM_3}{dt} = -cM_3 \cdot (\sigma_{\text{fission}}^3 + \sigma_\gamma^3) \cdot \Phi + c \{ M_3 \cdot (\sigma_{\text{fission}}^3 + \sigma_\gamma^3) + M_{5.9} \cdot (\sigma_{\text{fission}}^{5.9} + \sigma_\gamma^{5.8}) \} \cdot \Phi \cdot CR + M_3 \]

lost internally  \hspace{1cm} bred internally  \hspace{1cm} source

Example: If \( M_{5.9} = 0 \) and \( \frac{dM_3}{dt} = 0 \) and \( M_3 = 1090 \text{ (1-CR) kg} \text{ }^{233}\text{U}/\text{y} \), then

\[ c\Phi = \frac{M_3}{M_3 \cdot (\sigma_{\text{fission}}^3 + \sigma_\gamma^3) \cdot (1 - CR)} = \frac{1090 \text{ (1-CR) kg } / \text{ y}}{1500 \text{ kg } \cdot (531 + 46) \text{ } b \cdot (1 - CR)} = 0.001259 \text{ } b^{-1} \text{ y}^{-1} \]

(4)

From Eq. (3) with \( \frac{dM_3}{dt} = 0 \) after equilibrium is reached:

\[ M_3(t \to \infty) = \frac{M_3}{c \Phi \cdot (\sigma_{\text{fission}}^3 + \sigma_\gamma^3) \cdot (1 - CR)} = \frac{1090 \text{ kg } / \text{ y}}{0.001259 \text{ } b^{-1} \text{ y}^{-1} \cdot (531 + 46) \text{ } b} = 1500 \text{ kg} \]

\[ M_3(t = 0) = 1500 \text{ kg} \text{ is one possible initial condition.} \]

The following equation governs the concentrations of \(^{232}\text{U}\).

\[ \frac{dM_2}{dt} = -c \cdot M_2 \cdot \sigma_{\text{absorption}}^3 \cdot \Phi + M_{2_0} + c \cdot M_{1} \cdot (\sigma_\gamma^3) \cdot \Phi + k \cdot (M_3 \cdot \sigma_{\text{fission}}^3 + M_{5.9} \cdot \sigma_{\text{fission}}^{5.9}) \cdot c \cdot \Phi \cdot CR \text{ bred internally from } ^{231}\text{Pa} \]

lost internally  \hspace{1cm} source  \hspace{1cm} bred internally  \hspace{1cm} bred internally  \hspace{1cm} from fission \(^{235}\text{U} + ^{239}\text{Pu}\)

(5)

The ratio of atoms of \(^{232}\text{U}/^{233}\text{U} = F_{2.3} \) from the fusion breeder and is usually 2.4% to satisfy IAEA “self-protection,” but can be >5%.

\[ M_{2_0} = M_3 \cdot F_{2.3} \]

After a long time \( \frac{dM_2}{dt} = 0 \) \( M_5(t \to \infty) = M_9(t \to \infty) = 0 \)

\[ M_2(t \to \infty) = \frac{M_{2_0}}{c \cdot \sigma_{\text{absorption}}^3 \cdot \Phi} + M_1(t \to \infty) \cdot \frac{\sigma_\gamma^3}{\sigma_{\text{absorption}}^3} + k \cdot M_3(t \to \infty) \cdot CR \cdot \frac{\sigma_{\text{fission}}^3}{\sigma_{\text{absorption}}^3} \]

(6)

Plugging \( M_1(t \to \infty) \) from Eq (8) below, we get:

\[ M_2(t \to \infty) = \frac{1090 \text{ kg } / \text{ y} \cdot F_{2.3} \cdot (1 - CR)}{148 \cdot 0.001259 \text{ } b^{-1} \text{ y}^{-1}} + 3.97 \cdot M_{1_0} \cdot y \frac{200 \text{ } b}{148 \text{ } b} + k \cdot 1500 \text{ kg } \cdot CR \frac{531 \text{ } b}{148 \text{ } b} \]

\[ = 5849.8 \text{ } F_{2.3} \cdot (1 - CR) \text{ kg} + 5.365 \cdot M_{1_0} \cdot y + 5381.8 \text{ kg } \cdot CR \]
The following equation governs the concentrations of $^{231}$Pa. We assume an initial inventory from the fusion breeder of $^{231}$Pa in kg $M_1(t = 0)$ and an annual makeup of $\dot{M}_{10}$ in kg/y.

$$\frac{dM_1}{dt} = -cM_1 \cdot (\sigma_f + \sigma_{fission}) \cdot \Phi + \dot{M}_{10}$$

After a long time $\frac{dM_1}{dt} = 0$

$$M_1(t \to \infty) = \frac{\dot{M}_{10}}{c \cdot (\sigma_f + \sigma_{fission}) \cdot \Phi} = \frac{\dot{M}_{10}}{(200 + 0.02) \cdot 0.001259 \cdot b^{-1} \cdot y^{-1}} = 3.97 \dot{M}_{10} \text{y}$$

$$M_2(t \to \infty) = \frac{\dot{M}_{20}^2}{c \cdot \sigma_{absorption} \cdot \Phi} + M_1(t \to \infty) \cdot \frac{\sigma_f}{\sigma_{absorption}} + k \cdot M_3(t \to \infty) \cdot CR \cdot \frac{\sigma_{fission}}{\sigma_{absorption}}$$

$$M_3(t \to \infty) = \frac{\dot{M}_{30}}{c \cdot \sigma_{absorption} \cdot \Phi \cdot (\sigma_{fission} + \sigma_f) \cdot (1 - CR)}$$

neglecting $M_1$ and $k$ terms:

$$M_2(t \to \infty) = \frac{M_3 \cdot F_{2,3}}{M_3 \cdot \dot{M}_{30}} = \frac{F_{2,3} \cdot (\sigma_{fission} + \sigma_f) \cdot (1 - CR)}{\sigma_{absorption}} = \frac{F_{2,3} \cdot 577 b \cdot (1 - CR)}{148 b} = \frac{F_{2,3} \cdot 577 b \cdot (1 - CR)}{148 b} \cdot 3.90 \cdot F_{2,3} (1 - CR)$$

Including the M1 and k terms, the amount of $^{233}$U produced by breeding reactions in the MSR yearly is $M_3 \cdot (\sigma_{fission} + \sigma_f) \cdot c \cdot CR$ and contains a small amount of $^{232}$U, perhaps yearly 0.1%. The $^{233}$U supplier externally from a fusion breeder, $\dot{M}_3 \cdot (1 - CR)$ could come with up to 5% $^{232}$U. For this work lets assume supplied $^{233}$U has just enough $^{232}$U to be “self protected” by IAEA standards of 2.4%.³

Calculation of constant k used to calculate $^{232}$U: $\dot{M}_{10} = 0$ and CR=1;

$$\frac{M_2(t \to \infty)}{M_3(t \to \infty)} = k \cdot 3.58787 = 0.001 \quad k = 0.0002787$$
\[
\frac{M_2(t \to \infty)}{M_3(t \to \infty)} = \frac{5849.8 \cdot 0.024 \cdot (1 - CR) kg + 5.365 M_1 y + 0.0002787 \cdot 5381.8 kg \cdot CR}{1500 kg} \\
= 0.0936 \cdot (1 - CR) kg + 0.003577 \dot{M}_1 y / kg + 0.001 \cdot kg \cdot CR
\]

If CR=0, \( \dot{M}_1 = 0 \), \( F_{2,3} = 0.024 \) then \( \frac{M_2(t \to \infty)}{M_3(t \to \infty)} = 0.0936 \) The burnout rate of \( ^{232}U \) is slower than the resupply because the burnout rate of \( ^{233}U \) is faster and \( ^{232}U \) comes along with \( ^{233}U \).

A small fraction of fission neutrons have enough energy to produce \( ^{232}U \) from \( ^{232}Th \). We account for this production with the constant \( k \) adjusted to give \( ^{232}U / ^{233}U = 0.001 \).

**Startup and makeup \(^{233}U \) from a fusion breeder**

Using Eq. (10) we give an example where we supply initial fuel and make up from the fusion breeder with \( F_{2,3} = \frac{^{232}U / ^{233}U}{0.024} \) and the supplied \( ^{231}Pa = \dot{M}_1 = 0 \). The results are plotted in Fig. 1.

![Graph](image)

**Fig. 1. Ratio of \( ^{232}U / ^{233}U \) in steady state for makeup \( ^{233}U \) provided with \( ^{232}U / ^{233}U = 0.024 \).**

Below CR=0.7436 the steady state ratio of \( ^{232}U / ^{233}U \) is considerably above the 2.4% amount required to meet the IAEA “self-protection” standard even though the initial inventory and makeup is 2.4%. The reason for this enhancement of \( ^{232}U / ^{233}U \) is because the cross sections for destruction of \( ^{233}U \) is 577 barns whereas the cross sections for destruction of \( ^{232}U \) is only 148 barns as noted in Eq. 9. At a conversion ratio of 0.7436 the supplied \( ^{232}U \) just keeps up with its destruction. Above CR=0.7436 the ratio of \( ^{232}U / ^{233}U \) falls below the self-protection required amount of 2.4%. By supplying \( ^{231}Pa \) every year the amount of \( ^{232}U \) can be maintain at 2.4% as will be discussed next.
When $^{232}\text{U}/^{233}\text{U}$ is less than 0.024 we calculate the supply of $^{231}\text{Pa}$ needed to maintain this ratio. $\dot{M}_{10}$ should not be allowed to go negative. Eq. 10 becomes:

$$\begin{align*}
M_2(t \to \infty) &= 0.024 = 0.0936 \cdot (1 - CR) \text{ kg} + 0.003577 M_{10} \text{ y/ kg} + 0.001 \cdot \text{ kg} \cdot CR \\
\dot{M}_{10} &= \frac{-0.0936 \cdot (1 - CR) \text{ kg} - 0.001 \cdot CR + 0.024}{26.167 - 0.003577} \\
M_{10} &= -19.457 + CR \cdot 25.886; \quad M_{10} = 6.429 \text{ kg }^{231}\text{Pa/y} \text{ for } CR = 1
\end{align*}$$ (11)

Amount of $^{231}\text{Pa}$ needed to maintain “self-protection.”

Besides the annual supply of $^{231}\text{Pa}$ we need to have an initial inventory of $^{231}\text{Pa}$ for conversion ratio above 0.75. From Eq. 8 we get:

$$\begin{align*}
M_1(t \to \infty) &= 3.97 M_{10} = 3.97 \cdot (-19.457 + CR \cdot 25.886) \\
M_1(t \to \infty) &= -77.244 + 102.767CR \text{ for } CR \leq 0.7516 = 25.52 \text{ kg }^{231}\text{Pa} \text{ for } CR = 1
\end{align*}$$

The results of Eq (10) and (12) are plotted in Fig. 2.

![Fig. 2. Ratio of $^{232}\text{U}/^{233}\text{U}$ in steady state for makeup provided with $^{232}\text{U}/^{233}\text{U}=0.024$ and $^{231}\text{Pa}$ supplied yearly as shown. The inventory of $^{231}\text{Pa}$ is 3.97 times the yearly rate.](image)

In summary, starting up the MSR on fusion bred $^{233}\text{U}$ with 2.4% $^{232}\text{U}$ and supplying makeup fuel can satisfy “self-protection” up to a conversion ratio of 0.75. Above 0.75 we can still satisfy “self-protection” if we provide an initial inventory and makeup of $^{231}\text{Pa}$. 
Example: For the Li/MS fusion breeder design, we get 50 kg $^{231}$Pa per 1000 MW$_{\text{fusion}}$ per year (1300 MW$_{\text{nuclear}}$). 2500 MW$_{\text{nuclear}}$ fusion breeder would produce 96 kg $^{231}$Pa per full power year. Therefore the fusion breeder could supply 6.429 kg/y $^{231}$Pa to keep 15, 1000 MWe (2500 MW$_{\text{nuclear}}$) molten salt reactors “self-protected” for CR=1 and proportionally more as the conversion ratio is lower.

Other blanket examples are discussed in Ref. 4 — some with far more $^{231}$Pa production at the expense of $^{235}$U production.

**Reactor-grade Pu start up**

Suppose we start up the reactor on enough reactor grade Pu for criticality. Assume we provide an initial inventory of $^{231}$Pa and an annual amount so that the produced $^{233}$U is immediately provided with enough $^{232}$U to be “self protected.” When CR<1, makeup is $^{233}$U +2.4% $^{232}$U from the fusion breeder.

At t=0; M2=M3=0; M9=1500 kg $^{239}$Pu, $\dot{M}_3$ 0 = 1090 kg $^{233}$U(1−CR)/y

$M_1 = 0$ for CR < 0.7516

$M_1 = -77.244 + 102.767 \cdot CR$ for CR ≥ 0.7516

= 25.52 kg $^{231}$Pa for CR = 1

$M_{1_0} = -19.457 + CR \cdot 25.886$ for CR ≥ 0.7516

= 6.429 kg/y $^{231}$Pa for CR = 1

Eq. (1, 3, 5 and 7) become:

$$\frac{dM_9}{dt} = -cM_9 \cdot (\sigma^9_{\text{fission}} + \sigma^9_{\gamma}) \cdot \Phi = -0.001259 \cdot M_9 \cdot 1021 \cdot b = -1.2854 \cdot M_9$$ (13)

$$\frac{dM_3}{dt} = -cM_3 \cdot (\sigma^3_{\text{fission}} + \sigma^3_{\gamma}) \cdot \Phi + c(M_3 \cdot (\sigma^3_{\text{fission}} + \sigma^3_{\gamma}) + M_9 \cdot (\sigma^9_{\text{fission}} + \sigma^9_{\gamma})) \cdot \Phi \cdot CR + \dot{M}_3$$

$$= -0.001259 \cdot M_3 \cdot 577 \cdot (1−CR) + M_9 \cdot 0.001259 \cdot 1091 \cdot CR + 1090 \cdot (1−CR)$$

$$= -0.7264 \cdot M_3 \cdot (1−CR) + M_9 \cdot 1.285 \cdot CR + 1090 \cdot (1−CR)$$ (14)

\[
\frac{dM_2}{dt} = -c \cdot M_2 \cdot \sigma_{\text{absorption}} \cdot \Phi + M_2 \cdot \dot{\Phi} + \Phi \cdot M_1 \cdot (\sigma_1^t) \cdot \Phi + k \cdot (M_3 \cdot \sigma_{\text{fission}}^3 + M_9 \cdot \sigma_{\text{fission}}^9) \cdot \Phi \cdot CR
\]
\[
= -0.001259 \cdot 148 \cdot M_2 + 1090 \cdot (1 - CR) \cdot 0.024 + 0.001259 \cdot 200 \cdot M_1
\]
\[
+ 0.0002787 \cdot (M_3 \cdot 531 + M_9 \cdot 750) \cdot 0.001259 \cdot CR
\]
\[
= -0.18633 \cdot M_2 + 26.16 \cdot (1 - CR) + 0.2518 \cdot M_1 + (M_3 \cdot 0.0001863 + M_9 \cdot 0.0002632) \cdot CR
\]

(15)

\[
\frac{dM_1}{dt} = -cM_1 \cdot (\sigma_1^t + \sigma_{\text{fission}}^1) \cdot \Phi + \dot{M}_1
\]
\[
\frac{dM_1}{dt} = -0.001259 \cdot M_1 \cdot 200.02 \cdot b + \dot{M}_1
\]
\[
\quad \quad \quad \quad \quad = 0.2518 \cdot M_1 + \dot{M}_1
\]
\[
\dot{M}_1 = 0 \text{ for } CR < 0.7516
\]
\[
\quad \quad \quad \quad \quad = -19.457 + CR \cdot 25.886 \text{ for } CR \geq 0.7516
\]

(16)

The results are plotted in Fig. 3 and 4.

![Graph](image)

Fig. 3. The inventory buildup of $^{233}$U is shown with $^{239}$Pu startup.

In less than 2 years the $^{239}$Pu is replaced by $^{233}$U. The reactor model used here is sufficiently crude that the final inventory of $^{233}$U is different for various values of conversion ratio as can be seen in Fig. 3.
In Fig. 4 we see the ratio $^{232}\text{U}/^{233}\text{U}$ is below 0.024, the “self-protected” amount, up to over 10 years. Clearly an initial source of $^{232}\text{U}$ will be needed for startup on $^{239}\text{Pu}$ to keep $^{232}\text{U}/^{233}\text{U} = 0.024$. A possible solution to this problem will be discussed next.

**Alternative startup strategies**

If we tried to start up with $^{235}\text{U}$ we would need to add $^{238}\text{U}$ to denature below 20% or add $^{232}\text{U}$ to make “self-protected.” We do not know how to separate $^{232}\text{U}$ economically so this option appears unfeasible. Fully enriched $^{235}\text{U}$ would give startup similar to that of Fig. 3 and 4 but with a proliferation problem. An alternative might be considered using fusion produced $^{233}\text{U}$ with $>2.4\%$ $^{232}\text{U}$ to blend with the reactor grade Pu.

As an example lets look at startup with a guessed at initial conditions for $M_3=M_9=750 \text{ kg}$ and $M_2=5\%$ of $750=37.5 \text{ kg}$. $M_3_0 = 1090 \text{ kg }^{233}\text{U}(1-CR) / y$,

$$M_1_0 = 0 \text{ for } CR < 0.7516$$
$$= -19.457 + CR \cdot 25.886 \text{ for } CR \geq 0.7516$$

and

$$M_2_0 = 0.05 \cdot M_3_0$$

The results are plotted in Fig. 5 and 6. For the first year there is about twice as much $^{232}\text{U}/^{233}\text{U}$ than needed but in about a year the ratio settles down to 0.024. The excess $^{232}\text{U}$ is only 1.4 kg over the steady 36 kg. The guess was pretty good.
The problem of having the ratio of $^{232}\text{U}/^{233}\text{U}$ being below 2.4% for ten years of the all $^{239}\text{Pu}$ startup case was solved.

$^{230}\text{Th}$ from either fusion breeder or mining

$^{230}\text{Th}$ can be produced in a fusion breeder by the $^{232}\text{Th}(n,3n)^{230}\text{Th}$ reaction or by extracting from mined deposits where $^{230}\text{Th}$ is in secular equilibrium with $^{238}\text{U}$ at 17 ppm. Both sources are discussed in Ref 4. Initial inventories of $^{232}\text{U}$ and $^{233}\text{U}$ and annual makeup from fusion of $^{232}\text{U}$, $^{233}\text{U}$ and $^{231}\text{Pa}$ can allow startup and operation as shown in
Fig. 5 and 6. However, a source of $^{230}\text{Th}$ from mining could eliminate the need for fusion to sustain the reactor and maintain the “self-protection” feature of $^{232}\text{U}/^{233}\text{U}>2.4\%$. An initial inventory of 220 kg $^{230}\text{Th}$ and annual makeup of $^{230}\text{Th}=25.887\times\text{CR}-19.457$ for 1 GWe full power year assuming the makeup of fusion produced $^{233}\text{U}$ with $2.4\%^{232}\text{U}$. For $\text{CR}=1$ or $\text{CR}<1$ cases with no $^{232}\text{U}$ added such as $^{239}\text{Pu}$ makeup then we need 6.5 kg $^{230}\text{Th}$ per year. The annual $^{232}\text{Th}$ makeup is 1090 kg so the ratio of $^{230}\text{Th}/^{232}\text{Th}=0.006$. The initial inventory of 220 kg $^{230}\text{Th}$ for 117,000 kg of $^{232}\text{Th}$ gives $^{230}\text{Th}/^{232}\text{Th}=0.0019$.

**Two-fluid MSR**

The two-fluid MSR has a problem with making $^{233}\text{U}$ from Th in the blanket around the core or in pipes within the core, that is the second zone. There would ordinarily only be about $0.1\%^{232}\text{U}$ in the $^{233}\text{U}$.

This problem can be alleviated with a supply of $^{231}\text{Pa}$ from the fusion breeder placed in the second zone. The $^{232}\text{U}$ produced can be $2.4\%$ of the $^{233}\text{U}$ with production numbers the same as the one-zone results presented earlier. However, the amount of $^{232}\text{U}$ needed to maintain the ratio at $2.4\%$ in the core is far less than $2.4\%$ production in the $2^{nd}$ fluid. The amount of $^{232}\text{U}$ produced in the $2^{nd}$ fluid is far less than $2.4\%$ of $^{233}\text{U}$ (typically $\sim 0.1\%$), therefore this becomes a source of unprotected $^{233}\text{U}$ and is a proliferation concern. If the two-fluid MSR is pursued this problem needs to be further studied. Supplying an initial inventory of $^{232}\text{U}$ at startup for the two-fluid design remains essentially the same as for the one-fluid design. An initial inventory of $^{232}\text{U}$ can similarly come from the fusion breeder when it becomes commercially available.

**Value of $^{231}\text{Pa}$ and $^{233}\text{U}$**

Arguably an MSR can be designed with isobreeding of $^{233}\text{U}$ (CR=1) with no annual make up $^{233}\text{U}$ needed, just 6.4 kg/y and an initial inventory of 25.5 kg of $^{231}\text{Pa}$ for 1000 MWe to maintain proliferation resistance of $^{233}\text{U}$ at $^{232}\text{U}/^{233}\text{U}=0.024$. Otherwise the MSR would be designed for CR=0.8 with annually supplied $^{233}\text{U}$ of $1090\times(1-0.8)=218$ kg/y. This implies we can “afford” to pay up to 218/6.4=34.1 time as much per kg for $^{231}\text{Pa}$ as for $^{233}\text{U}$. $60\$/g $^{233}\text{U}$ implies $2050\$/g $^{231}\text{Pa}$ if the entire savings were applied to the value of $^{231}\text{Pa}$. The actual price might be well below, perhaps 50\% or about 1000 $/g $^{231}\text{Pa}$ owing to other considerations. A direct consequence is a fusion breeder can cost considerably more if it produces $^{231}\text{Pa}$ than if it just produces $^{233}\text{U}$ if the above arguments are valid and a strong demand develops to create a market for the fusion breeder and its produced isotopes, $^{233}\text{U}$, $^{232}\text{U}$ and $^{231}\text{Pa}$. From Ref. 4 we show the cost of producing $^{233}\text{U}$ falls below 60$/g for Q>3.

An estimate for the value of $^{233}\text{U}$ could be derived from the cost of mined and enriched $^{235}\text{U}$ with a correction for the differences in utilization of the two. However, $^{233}\text{U}$ coming
from fusion has the distinction of having $^{232}\text{U}$ that might allow the equivalent of high enrichment because of its self-protection. In this sense, $^{233}\text{U}$ plus $2.4\%^{232}\text{U}$ is likely much more valuable than $^{235}\text{U}$ if this “high enrichment” becomes acceptable and if a market for a reactor that utilizes it comes along and if the fusion breeder becomes commercial.

The cost of the 218 kg/y of $^{233}\text{U}$ at 60$/g per unit of power is:

$$\frac{218\text{ kg} \times 60,000\text{ $/kg}}{10^6\text{ kWe} \cdot \text{y} \times 8766\text{ h} / \text{y}} = \frac{13.08 \times 10^6\text{ $}}{10^6\text{ kWe} \cdot 8766\text{ h}} = 0.00149\text{ $/kWeh}$$

The cost of the 6.4 kg/y of $^{231}\text{Pa}$ at 2050$/g per unit of power is:

$$\frac{6.4\text{ kg} \times 2050,000\text{ $/kg}}{10^6\text{ kWe} \cdot \text{y} \times 8766\text{ h} / \text{y}} = \frac{13.12 \times 10^6\text{ $}}{10^6\text{ kWe} \cdot 8766\text{ h}} = 0.001497\text{ $/kWeh}$$

For comparison, the cost of power is typically 0.05 $/kWeh. This means the savings from breeding all the MSR’s own fuel is 0.0015/0.05=2.3%, that is, relatively a small amount and supplying a modest amount of annual makeup $^{233}\text{U} + 2.4\%^{232}\text{U}$ is easily doable once the fusion breeder becomes available commercially.

**Conclusion**

When fusion technology becomes commercially available at modest performance expressed by $Q\sim3$ ($Q=$fusion power/input power) molten salt reactors could be provided initial inventories and make up $^{233}\text{U}$ fuel with enough $^{232}\text{U}$ to satisfy IAEA “self-protection” for nonproliferation, that is 2.4%. This result comes from fusion’s extra high energy neutron being able to make large amounts of $^{233}\text{U}$ with enough $^{232}\text{U}$. Startup on reactor grade $^{239}\text{Pu}$ falls short in the first ten years to meet the criterion of $2.4\%^{232}\text{U}$ in $\text{U}$ but startup on 50% $^{233}\text{U}$ and 50% Pu works well in satisfying the criterion. Molten salt reactors should be able to be designed to produce most of if not all their own makeup fuel and remain “self-protected” with a small inventory and make up of $^{231}\text{Pa}$ also supplied by the fusion breeder. As the conversion ratio approaches unity the necessary $^{231}\text{Pa}$ can be supplies from one fusion breeder to keep 15 molten salt reactors self-protected of equal nuclear power for one specific case examined. $^{230}\text{Th}$ from mining could sustain the reactor and not need fusion to maintain nonproliferation but it would still be required for startup. An interesting question becomes, when will adequate fusion technology become available and will interest in fission power be strong enough to form enough demand to take advantage of this technology?